Hydraulics

Solution Sheet 1 – Drag on bodies

1. Discussion in class: *What effects does gravity have on the nature of "internal" flows such as those we have been considering – flow past bluff bodies etc, where there is no free surface?*

2. *The drag coefficient of a wide plate normal to a flow is approximately $C_\mathrm{D} \approx 2$. Let the free stream conditions be U_0 and p_0. If the average pressure on the front of the plate is approximately equal to the stagnation pressure $p_0 + \frac{1}{2}\rho U_0^2$, what is the average pressure on the rear?*

   In the case of a flat plate aligned normally to the flow, there is little viscous drag, and the force is predominantly just due to the difference in pressure force between front and rear. Hence if A is the plate area, we have

   $$\left(p_0 + \tfrac{1}{2}\rho U_0^2\right) A - p_\mathrm{rear} A = \tfrac{1}{2}\rho C_\mathrm{D} A U_0^2,$$

   giving

   $$p_\mathrm{rear} = p_0 + \tfrac{1}{2}\rho U_0^2 - \tfrac{1}{2}\rho \times 2.0 \times U_0^2 \approx p_0 - \tfrac{1}{2}\rho U_0^2.$$

   Hence, rather less than ambient pressure!

3. *A cylindrical chimney is $2\,\mathrm{m}$ in diameter and $40\,\mathrm{m}$ high. When it is subject to an $80\,\mathrm{km\,h^{-1}}$ storm wind, what is the force on it, and where does it occur? (The drag coefficient of a cylinder can vary between 0.3 and 1, depending on the Reynolds number; $\rho_\mathrm{air} = 1.2\,\mathrm{kg\,m^{-3}}$ at $20\,^\circ\mathrm{C}$).*

   In this question we ignore the fact that the wind actually forms a turbulent boundary layer on the earth so that we assume that there is no variation in wind speed with height. Hence

   $$\begin{aligned} F &= \tfrac{1}{2}\rho C_\mathrm{D} A U_0^2 \\ &= \tfrac{1}{2} \times 1.2 \times 1 \times 2 \times 40 \times \left(80 \times \frac{1000}{3600}\right)^2 \\ &\approx 24000\,\mathrm{N}, \\ &\quad \text{while for } C_\mathrm{D} = 0.3 \text{ this is } 7200\,\mathrm{N} \end{aligned}$$

   In this case, as we have assumed that velocity is constant along the chminey, the force occurs halfway up it. In problems where there is a more complicated pressure variation, this is, of course, no longer the case, and we find the position by taking moments.

4. *A pizza delivery vehicle has a long thin rectangular sign on top aligned with the direction of travel. If the car travels at $50\,\mathrm{km\,h^{-1}}$, estimate (a) the force on the sign with no crosswind, and (b) discuss the effect of a crosswind.*

   The drag force on a body in a two-dimensional flow is given by $U\sqrt{U^2+V^2}$, where U is the velocity in the direction of interest and V is the component perpendicular to that. To put numbers on this, let us assume that the "long thin rectangular sign" is $1\,\mathrm{m}$ high, $2\,\mathrm{m}$ long and $2\,\mathrm{cm}$ thick. We will assume that the drag coefficient in both directions is the same.

x-Force on moving car at $50\,\mathrm{km\,h^{-1}}$, no side wind	$=$	$\frac{1}{2}\rho C_\mathrm{D} \times 0.02 \times 1 \times \left(50 \times \frac{1000}{3600}\right)^2$
	$\propto$	$0.02 \times 1 \times 50^2 = 50$
y-Force on stationary car with side wind	$\propto$	$1 \times 2 \times 20^2 = 800$
x-Force on moving car with side wind	$\propto$	$0.02 \times 1 \times 50\sqrt{50^2+20^2} \approx 54$
y-Force on moving car with side wind	$\propto$	$1 \times 2 \times 20\sqrt{50^2+20^2} \approx 2150$!!!

   We can see that the lateral force has been hugely increased by adding on the velocity of the vehicle.

   What lessons might we learn from this?

   a. Drivers of pizza delivery vehicles should be careful in high winds.

   b. Drivers of large long lorries on highways should be *particularly* careful in windy weather.

c. We should not be surprised when we see TV films after a storm of highways with overturned lorries.